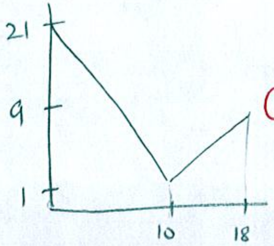
A person's velocity (in meters per minute) at time t (in minutes) is given by $v(t) = \begin{cases} 21-2t, & 0 \le t \le 10 \\ t-9, & 10 \le t \le 18 \end{cases}$. **SCORE: _____ / 5 PTS**

[a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 18$ seconds.
NOTE: You must show the arithmetic expression that you used to get your answer.



$\frac{1}{2}(21+1)(10-0) + \frac{1}{2}(1+9)(18-10)$

$= \frac{1}{2}(21+1)(10) + \frac{1}{2}(1+9)(8)$ 1 1

$= 110 + 40$

$= 150$m $\frac{1}{2}$

[b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 18$ seconds using three subintervals and right endpoints.
NOTE: You must show the arithmetic expression that you used to get your answer.

$\Delta t = \frac{18-0}{3} = 6$

$v(6)\Delta t + v(12)\Delta t + v(18)\Delta t$

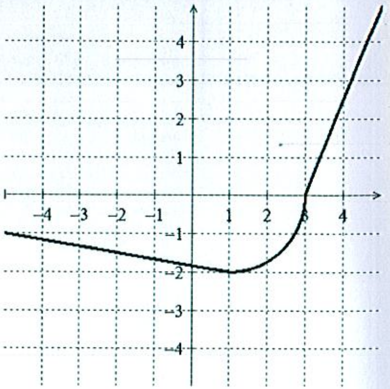
$= (9 + 3 + 9)(6)$ 2

$= 126$m $\frac{1}{2}$

The graph of function f is shown on the right.

SCORE: _____ / 4 PTS

<u>The graph consists of a diagonal line, an arc of a circle, then another diagonal line.</u>



[a] Evaluate $\int_{-5}^{5} f(x)\,dx$.

<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>

1 $\underbrace{-\frac{1}{2}(1+2)(6)}\ \underbrace{-\frac{1}{4}\pi(2)^2}_{1}\ \underbrace{+\frac{1}{2}(2)(5)}$ $\frac{1}{2}$

$= \underbrace{-4-\pi}_{\frac{1}{2}}$

[b] Evaluate $\int_{5}^{1} f(x)\,dx$.

$= -\int_1^5 f(x)\,dx = -\left[-\frac{1}{4}\pi(2)^2 + \frac{1}{2}(2)(5)\right] = \underline{\pi - 5}$ 1

NO POINTS FOR $5-\pi$

Using the limit definition of the definite integral, and right endpoints, find $\int_{-1}^{3}(3x^2 - x - 4)\,dx$.

SCORE: _____ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{3-^-1}{n} = \frac{4}{n}$$

$$1 \quad \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\frac{4i}{n}\right)\frac{4}{n}$$

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left[3\left(-1+\frac{4i}{n}\right)^2 - \left(-1+\frac{4i}{n}\right) - 4\right] \quad 2$$

1

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left(\frac{-24i}{n} + \frac{48i^2}{n^2} - \frac{4i}{n}\right)$$

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left(\frac{-28i}{n} + \frac{48i^2}{n^2}\right) \quad 2$$

$$= \lim_{n\to\infty}\frac{4}{n}\left(-\frac{28}{n}\sum_{i=1}^{n} i + \frac{48}{n^2}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty}\frac{4}{\cancel{n}}\left(-\frac{\overset{14}{\cancel{28}}}{n}\,\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{\overset{8}{\cancel{48}}}{n^2}\,\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right)$$

1   1

$$= 4(-14+16)$$

$$= 8 \quad 1$$

1 FOR HAVING $\lim_{n\to\infty}$ ON EACH LINE THAT STILL INVOLVES "n"

Evaluate $\int_{-6}^{6} (|x-2| - 3\sqrt{36-x^2})\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

$$= \underbrace{\int_{-6}^{6} |x-2|\,dx - 3\int_{-6}^{6} \sqrt{36-x^2}\,dx}_{2} = \underbrace{\tfrac{1}{2}(8)(8) + \tfrac{1}{2}(4)(4)}_{1} \underbrace{- 3\left(\tfrac{1}{2}\pi(6)^2\right)}_{1}$$

$$= \underbrace{40 - 54\pi}_{1}$$